Math. 152, Problems for the streaming review for exam 1.

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1. Set an integral for each of the volumes of the solids obtained by rotating the region enclosed by the graphs of \( f(x) = x^2 \) and \( g(x) = 2 - x^2 \) about the following axes:
   (a) The \( x \) axis.  
   (b) The line \( y = -1 \).  
   (c) The line \( x = 2 \).

2. Let \( S \) be the area between the lines \( y = 2x, x = 5 \) and above the \( x \)-axis. Set an integral for the volume that its base is the area \( S \) and its cross sections that perpendicular to the \( y \)-axis are squares.

3. Evaluate each of the following indefinite integrals.
   (a) \( \int x^6 \ln x \, dx \)  
   (b) \( \int \frac{dx}{x^3 \sqrt{x^2 - 4}} \)  
   (c) \( \int \frac{x}{x^2(x - 1)} \, dx \)  
   (d) \( \int \tan^3 x \sec^4 x \, dx \) in two ways.

4. Evaluate each of the following definite integrals.
   (a) \( \int_0^{\pi/4} \sin^3 x \sqrt{x} \, dx \)  
   (b) \( \int_0^{\pi} x \sin x \, dx \)  
   (c) \( \int_0^1 e^{x+e^x} \, dx \)

5. (a) Find \( A \), the average value of the function \( f(x) = e^{\sqrt{x}} \) on the interval \([0, 4]\).
   (b) For what value of \( x \) the function gets its average \( A \)?

6. Let \( I = \int_0^1 e^{-\frac{x^2}{2}} \, dx \)
   (a) Find the error bound when approximating I using the Trapezoidal Method with \( N = 100 \).
   (b) Find the value of \( N \) for which the error when approximating I using the Trapezoidal Method is less than \( 10^{-6} \).