Your first midterm examination is likely to contain some problems that do not resemble these review problems.

I. Applications of Integration

(1-3) Find the volumes of the solids obtained by rotating the indicated region \( \mathcal{R} \) in the \( xy \)-plane about the specified axis:

1. Region: \( \mathcal{R} \) is bounded by \( y = 1, \ y = \ln x \) and \( x = e^2 \).
   - Axes: (a) the line \( y = -1 \); (b) the line \( x = -2 \).

2. Region: \( \mathcal{R} \) consists of all points \((x, y)\) with \(0 \leq x \leq \pi\) and \(0 \leq y \leq \sin x\).
   - Axes: (2a) the line \( y = -2 \); (2b) the line \( x = -1 \).

3. Region: \( \mathcal{R} \) is bounded by \( x = y(4 - y) \) and the \( y \)-axis
   - Axis: the \( y \)-axis.

4. Write down the integral used to compute the work done against gravity in building a granite pyramid 500 feet high with a square base of side length 800 feet. Take the density of granite as 170 lbs per cubic foot. Explain in detail how the integral is obtained.

5. There is a point \( x_0 \) in the interval \([5,7]\) where the function \( f(x) = (x^2 - 4)^{-1/2} \) takes on its average value over that interval.
   - (a) How do we know that? (b) Find such a point \( x_0 \).

II. Numerical Methods

(6) How many subintervals of \([0,2]\) should we use to ensure an accuracy within \(10^{-6}\) when we approximate \( \int_{0}^{2} 4x^3 - x^4 \, dx \) using:
   - (a) the Midpoint Rule?; (b) Simpson’s Rule?

(7) A certain integral \( \int_{1}^{4} f(x) \, dx \) is approximated by the Trapezoidal Rule using 30 intervals, and the approximation found is 3.14286. The graph of \( f''(x) \) is shown here. Find a range of values \([a,b]\) such that the exact value of the integral can be guaranteed to lie within that range, and explain your method. (The numbers \( a \) and \( b \) should be given to at most 3 decimal places accuracy.)

Figure 1: \( f'' \)
III. Techniques of Integration

(8) Evaluate the following integrals.

(a) \( \int \sin^3 x \cos^4 x \, dx \) (b) \( \int \sec^4 x \, dx \)

(c) \( \int \tan^5 x \sec^3 x \, dx \) (d) \( \int \sec^3 x \, dx \)

(9) Evaluate the following integrals. Integral (g) is very difficult unless you are given the following hint: The function \( \frac{1}{1+e^x} - \frac{1}{2} \) is an odd function.

(a) \( \int x^5 (\ln x)^2 \, dx \) (b) \( \int \frac{dx}{x \ln x} \) (c) \( \int \cos (\sqrt{x}) \, dx \)

(d) \( \int x^2 \tan^{-1} x \, dx \) (e) \( \int x^{-2} \sin^{-1} x \, dx \) (f) \( \int e^{\sqrt{x}} \, dx \)

(g) \( \int_{\pi/2}^{\pi/2} \frac{\cos(x)}{1 + e^x} \, dx \)

(10) Evaluate the following integrals.

(a) \( \int \frac{dx}{(25 + x^2)^2} \) (b) \( \int \frac{x \, dx}{(x^2 + 36)(x + 1)} \) (c) \( \int \frac{dx}{\sqrt{2x - x^2}} \)

(d) \( \int \frac{x \, dx}{(x - 5)(x + 3)^2} \) (e) \( \int \frac{x^2 \, dx}{(16 - x^2)^{3/2}} \) (f) \( \int \frac{dx}{x^2 + 4x + 9} \)

(11) Evaluate \( \int \sin(\ln x) \, dx \) using two integrations by parts. Would another method work?