Your first midterm examination is likely to contain some problems that do not resemble these review problems.

I. Applications of Integration

1-3) Find the volumes of the solids obtained by rotating the indicated region \( \mathcal{R} \) in the \( xy \)-plane about the specified axis:

(1) Region: \( \mathcal{R} \) is bounded by \( y = 1 \), \( y = \ln x \) and \( x = e^2 \).
   Axes: (a) the line \( y = -1 \); (b) the line \( x = -2 \).

(2) Region: \( \mathcal{R} \) consists of all points \((x, y)\) with \( 0 \leq x \leq \pi \) and \( 0 \leq y \leq \sin x \).
   Axes: (2a) the line \( y = -2 \); (2b) the line \( x = -1 \).

(3) Region: \( \mathcal{R} \) is bounded by \( x = y(4 - y) \) and the \( y \)-axis
   Axis: the \( y \)-axis.

(4) Write down the integral used to compute the work done against gravity in building a granite pyramid 500 feet high with a square base of side length 800 feet. Take the density of granite as 170 lbs per cubic foot. Explain in detail how the integral is obtained.

(5) There is a point in the interval \([5, 7]\) where the function \( f(x) = (x^2 - 4)^{-1/2} \) takes on its average value over that interval.
   (a) How do we know that? (b) Find such a point.

II. Numerical Methods

(6) How many subintervals of \([0, 2]\) should we use to ensure an accuracy within \( 10^{-6} \) when we approximate \( \int_0^2 (4x^3 - x^4) \, dx \) using:
   (a) the Midpoint Rule?; (b) Simpson’s Rule?

(7) A certain integral \( \int_{-1}^1 f(x) \, dx \) is approximated by the Trapezoidal Rule using 30 intervals, and the approximation found is 3.14286. The graph of \( f''(x) \) is shown here. Find a range of values \([a, b]\) such that the exact value of the integral can be guaranteed to lie within that range, and explain your method. (The numbers \( a \) and \( b \) should be given to at most 3 decimal places accuracy.)
III. Techniques of Integration

(8) Evaluate the following integrals.
   (a) \( \int \sin^3 x \cos^4 x \, dx \)   (b) \( \int \sec^4 x \, dx \)
   (c) \( \int \tan^5 x \sec^3 x \, dx \)   (d) \( \int \sec^3 x \, dx \)

(9) Evaluate the following integrals.
   (a) \( \int x^5 (\ln x)^2 \, dx \)   (b) \( \int \frac{dx}{x \ln x} \)   (c) \( \int \cos (\sqrt{x}) \, dx \)
   (d) \( \int x^2 \tan^{-1} x \, dx \)   (e) \( \int x^{-2} \sin^{-1} x \, dx \)   (f) \( \int e^{\sqrt{x}} \, dx \)
   (g) \( \int_{\pi/2}^{\pi} \cos(x) \, dx \)

(10) Evaluate the following integrals.
     (a) \( \int \frac{dx}{(25 + x^2)^2} \)   (b) \( \int \frac{x \, dx}{(x^2 + 36)(x + 1)} \)   (c) \( \int \frac{dx}{\sqrt{2x - x^2}} \)
     (d) \( \int \frac{x^2 - x + 4}{(x - 5)(x + 3)^2} \, dx \)   (e) \( \int \frac{x^2 \, dx}{(16 - x^2)^{3/2}} \)   (f) \( \int \frac{dx}{x^2 + 4x + 9} \)

(11) Evaluate \( \int \sin (\ln x) \, dx \) using two integrations by parts. Would another method work?