Math 152, Fall 2010, Formula Sheet for the Final Exam

\sin(0) = 0; \ \sin(\pi/6) = 1/2; \ \sin(\pi/4) = \sqrt{2}/2; \ \sin(\pi/3) = \sqrt{3}/2; \ \sin(\pi/2) = 1
\cos(0) = 1; \ \cos(\pi/6) = \sqrt{3}/2; \ \cos(\pi/4) = \sqrt{2}/2; \ \cos(\pi/3) = 1/2; \ \cos(\pi/2) = 0

\cos^2 x + \sin^2 x = 1; \ 1 + \tan^2 x = \sec^2 x; \ 1 + \cot^2 x = \csc^2 x

\sin(2x) = 2 \sin x \cos x; \ \cos(2x) = \cos^2 x - \sin^2 x

\cos^2 x = \frac{1}{2}(1 + \cos(2x)); \ \sin^2 x = \frac{1}{2}(1 - \cos(2x))

\sin A \cos B = \frac{1}{2} [\sin(A - B) + \sin(A + B)]
\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]
\cos A \cos B = \frac{1}{2} [\cos(A - B) + \cos(A + B)]

\int \sec x \, dx = \ln |\sec x + \tan x| + C; \ \int \csc x \, dx = -\ln |\csc x + \cot x| + C

If \( T_N, M_N, S_N \) are the Trapezoidal, Midpoint and Simpson’s approximations, then

\[ T_N = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{N-2}) + 2f(x_{N-1}) + f(x_N)]; \]
\[ M_N = \Delta x [f(c_1) + f(c_2) + \cdots + f(c_N)] \text{ where } c_j = (x_{j-1} + x_j)/2; \]
\[ S_N = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N)]. \]

If \( I = \int_a^b f(x) \, dx \) then

\[ |T_N - I| \leq \frac{K_2(b - a)^3}{12N^2}, \quad |M_N - I| \leq \frac{K_2(b - a)^3}{24N^2}, \quad |S_N - I| \leq \frac{K_4(b - a)^5}{180N^4}. \]

The length of the curve \( y = f(x), a \leq x \leq b \) is equal to \( \int_a^b \sqrt{1 + (f'(x))^2} \, dx \).

The area of the surface obtained by rotating the curve \( y = f(x), a \leq x \leq b \) about the \( x \)-axis is equal to \( \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} \, dx \).

The length of the parametric curve \( (x(t), y(t)), a \leq t \leq b \) equals \( \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} \, dt \).

If a curve is given in polar form by \( r = f(\theta) \) then the area bounded by \( r = f(\theta), \theta = \alpha \) and \( \theta = \beta \) is \( \frac{1}{2} \int_\alpha^\beta r^2 \, d\theta = \frac{1}{2} \int_\alpha^\beta (f(\theta))^2 \, d\theta \).

Newton’s Law of Cooling is given by \( \frac{dT}{dt} = -k(T - T_0) \), where \( T \) is the temperature and \( T_0 \) is the ambient temperature. The balance \( P \) in an annuity is given by \( \frac{dP}{dt} = r(P - N/r) \), where \( r \) is the interest rate and \( N \) is the rate of withdrawal.

The \( n \)th Taylor polynomial of \( f(x) \) with center \( a \) is

\[ T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} (x - a)^k \]

If \( |f^{(n+1)}(u)| \leq K \) for all \( u \) between \( a \) and \( x \), then

\[ |f(x) - T_n(x)| \leq K \frac{|x - a|^{n+1}}{(n + 1)!} \]

\((1 + x)^a = 1 + ax + \frac{a(a - 1)}{2!} x^2 + \frac{a(a - 1)(a - 2)}{3!} x^3 + \cdots \) if \( |x| < 1 \).