Math 152, Spring 2011, Formula Sheet for Exam 2

\[
\begin{align*}
\sin(0) &= 0 ; & \sin(\pi/6) &= 1/2 ; & \sin(\pi/4) &= \sqrt{2}/2 ; & \sin(\pi/3) &= \sqrt{3}/2 ; & \sin(\pi/2) &= 1 \\
\cos(0) &= 1 ; & \cos(\pi/6) &= \sqrt{3}/2 ; & \cos(\pi/4) &= \sqrt{2}/2 ; & \cos(\pi/3) &= 1/2 ; & \cos(\pi/2) &= 0 \\
\cos^2 x + \sin^2 x &= 1 ; & 1 + \tan^2 x &= \sec^2 x ; & 1 + \cot^2 x &= \csc^2 x \\
\sin(2x) &= 2 \sin x \cos x ; & \cos(2x) &= \cos^2 x - \sin^2 x \\
\cos^2 x &= \frac{1}{2} (1 + \cos(2x)) ; & \sin^2 x &= \frac{1}{2} (1 - \cos(2x)) \\
\sin A \cos B &= \frac{1}{2} \left[ \sin(A - B) + \sin(A + B) \right] \\
\sin A \sin B &= \frac{1}{2} \left[ \cos(A - B) - \cos(A + B) \right] \\
\cos A \cos B &= \frac{1}{2} \left[ \cos(A - B) + \cos(A + B) \right] \\
\int \sec x \, dx &= \ln |\sec x + \tan x| + C ; & \int \csc x \, dx &= -\ln |\csc x + \cot x| + C
\end{align*}
\]

If \( T_N, M_N, S_N \) are the Trapezoidal, Midpoint and Simpson’s approximations, then

\[
T_N = \frac{\Delta x}{2} \left[ f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{N-2}) + f(x_{N-1}) + f(x_N) \right] ;
\]

\[
M_N = \Delta x \left[ f(c_1) + f(c_2) + \cdots + f(c_N) \right] \text{ where } c_j = (x_{j-1} + x_j)/2 ;
\]

\[
S_N = \frac{\Delta x}{3} \left[ f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{N-2}) + 4f(x_{N-1}) + f(x_N) \right] .
\]

If \( I = \int_a^b f(x) \, dx \) then

\[
|T_N - I| \leq \frac{K_2(b - a)^3}{12N^2} , & |M_N - I| \leq \frac{K_2(b - a)^3}{24N^2} , & |S_N - I| \leq \frac{K_4(b - a)^5}{180N^4} .
\]

The length of the curve \( y = f(x) \), \( a \leq x \leq b \) is equal to \( \int_a^b \sqrt{1 + (f'(x))^2} \, dx \).

The area of the surface obtained by rotating the curve \( y = f(x) \), \( a \leq x \leq b \) about the x-axis is equal to \( \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} \, dx \).

The length of the parametric curve \( (x(t), y(t)) \), \( a \leq t \leq b \) equals \( \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} \, dt \).

If a curve is given in polar form by \( r = f(\theta) \) then the area bounded by \( r = f(\theta) \), \( \theta = \alpha \) and \( \theta = \beta \) is \( \int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta = \int_{\alpha}^{\beta} \frac{1}{2} (f(\theta))^2 d\theta \). The length of the polar curve \( r = f(\theta) \) between \( \theta = \alpha \) and \( \theta = \beta \) is \( \int_{\alpha}^{\beta} \sqrt{r^2 + (\frac{dr}{d\theta})^2} \, d\theta = \int_{\alpha}^{\beta} \sqrt{(f(\theta))^2 + (f'(\theta))^2} \, d\theta \).

Newton’s Law of Cooling is given by \( \frac{dT}{dt} = -k(T - T_0) \), where \( T \) is the temperature and \( T_0 \) is the ambient temperature. The balance \( P \) in an annuity is given by \( \frac{dP}{dt} = r(P - N/r) \), where \( r \) is the interest rate and \( N \) is the rate of withdrawal.

The nth Taylor polynomial of \( f(x) \) with center \( a \) is \( T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!} (x - a)^k \).

If \( |f^{(n+1)}(u)| \leq K \) for all \( u \) between \( a \) and \( x \), then \( |f(x) - T_n(x)| \leq K \frac{|x - a|^{n+1}}{(n+1)!} \).