Math 151, Spring 2009, Review Problems for Exam 1

Your first exam is likely to have problems that do not resemble these review problems.

Precalculus

1. Find the domain and range of the following functions.
   (a) $\sqrt{3 - x}$       (b) $\frac{1}{\sqrt{x^2 + 1}}$       (c) $\frac{1}{\sqrt{2 - x}}$

2. Express the set of real numbers $x$ satisfying the given condition as an interval.
   (a) $|x + 2| < 7$       (b) $|3x - 1| \geq 5$       (c) $|x + 2| < 7$ and $|3x - 1| \geq 5$ are true.

3. Find $(f \circ g)(x)$ and $(g \circ f)(x)$, where $f(x) = \sqrt{x^2 + 2}$ and $g(x) = x^2 + 1$.

4. Find the inverse of the function $f(x) = \frac{x}{x + 1}$.

5. Simplify $\cot(\sin^{-1}(x))$ and $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$.

Limits and Continuity

6. Find the following limits
   (a) $\lim_{x \to -2} \frac{\sqrt{x + 1} - 1}{x + 3}$       (b) $\lim_{x \to -3} \frac{x(x + 1)}{x + 3}$

7. Find the following limits
   (a) $\lim_{x \to -2} \frac{\sqrt{4x + 1} - 3}{x - 2}$       (b) $\lim_{x \to -5} \frac{x(x - 5)}{\sqrt{x} - \sqrt{5}}$       (c) $\lim_{x \to -2} \frac{x^2 + 4x - 12}{x^2 - 9x + 14}$

8. Show that the equation $x^2 - \cos(x) = 1$ has a solution in the open interval (1, 2).

9. Use the Squeeze Theorem to show that $\lim_{x \to 0} x^2 \sin\left(\frac{3}{x^3}\right) = 0$.

10. Find the following limits
    (a) $\lim_{x \to 0} \frac{\tan(3x^2)}{x^2}$       (b) $\lim_{x \to 0} \frac{\cos^2(x) - \cos(x)}{x}$       (c) $\lim_{x \to 0} \frac{\sin(4x)}{x}$
    (d) $\lim_{x \to 0} \frac{2x + 2 \sin(x) + 6 \cos(x) - 6}{3x}$

11. True or false? The following limit exists
    $$\lim_{x \to 5} \frac{|x - 5|}{x - 5}.$$
12. True or false? The function
\[
f(x) = \begin{cases} 
2x^2 + 1, & x > 3 \\
19, & x = 3 \\
5x + 4 + \sin(x - 3), & x < 3 
\end{cases}
\]
is continuous at \(x = 3\). Give details to justify your answer.

13. Find the following limits
(a) \(\lim_{x \to 4^-} \frac{4 - x}{|x - 4|}\) 
(b) \(\lim_{x \to 3^+} \frac{|x - 3|}{x - 3}\)

14. Find the values of \(a\) and \(b\) that will make the function
\[
f(x) = \begin{cases} 
x^2 + 1, & x < 1 \\
a x + b, & 1 \leq x \leq 2 \\
x^3, & x > 2 
\end{cases}
\]
continuous everywhere.

15. Use the \(\epsilon - \delta\) formal definition of the limit to prove that \(\lim_{x \to 2} 6x + 2 = 14\).

**Derivatives**

16. Use the limit definition of the derivative to compute \(f'(x)\) for \(f(x) = 2x^2 + 1\).

17. Do the following:
(a) Find the equation for the tangent line to the curve \(y = x^3 + x^2 + x + 1\) at the point \((1, 4)\).
(b) Find the equation for the line that also passes through \((1, 4)\), but is perpendicular to the line you found in (a).

18. Let \(f(x) = x + \frac{1}{x}\). Find all the points on the graph \(f(x)\) where the tangent line is horizontal.

19. Find the derivative of each function.
(a) \((2x + 1)^3 e^{2x}\) 
(b) \(\frac{x^2 + x + 1}{\sin(2x)}\) 
(c) \(\tan(x^3 + 3x + 1)\)

20. Find the second derivative of each function.
(a) \(\frac{(x^2 + 1)^{20}}{x + 1}\) 
(b) \(\frac{e^x}{x + 1}\) 
(c) \(xe^{x^2}\)

21. An object is moving along the \(x\)-axis and its position at any time \(t \geq 0\) is given by \(x(t) = -2t^3 + 3t\). Find the velocity and acceleration of the object at \(t = 1\). Is the object moving forward or backwards at \(t = 1\)? Is it speeding up or slowing down at \(t = 1\)?

22. Assume that \(f(2) = 3, f'(2) = -1, g(2) = -2, g'(2) = 6, f'(-2) = -2,\) and \(g'(3) = 4\). Use this information to calculate \((f \circ g)'(2)\) and \((g \circ f)'(2)\).