Lines: If \((x_1, y_1), (x_2, y_2)\) lie on a line \(L\), the slope of \(L\) is \(m = \frac{y_2 - y_1}{x_2 - x_1}\) and the equation is \(y - y_1 = m(x - x_1)\).

Distance, \((x_1, y_1)\) to \((x_2, y_2)\): \(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\).

Circle, center \((a, b)\), rad. \(r\): \((x - a)^2 + (y - b)^2 = r^2\).

Trig: In a right triangle:
- \(\sin \theta = \frac{\text{opp}}{\text{hyp}}\)
- \(\cos \theta = \frac{\text{adj}}{\text{hyp}}\)
- \(\tan \theta = \frac{\text{opp}}{\text{adj}}\)
- \(\cot \theta = \frac{\text{adj}}{\text{opp}}\)
- \(\sec \theta = \frac{1}{\cos \theta}\)
- \(\csc \theta = \frac{1}{\sin \theta}\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(0)</th>
<th>(\pi/6)</th>
<th>(\pi/4)</th>
<th>(\pi/3)</th>
<th>(\pi/2)</th>
<th>(\pi)</th>
<th>(3\pi/2)</th>
<th>(2\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin x)</td>
<td>0</td>
<td>1/2</td>
<td>(\sqrt{3}/2)</td>
<td>0</td>
<td>(\sqrt{4}/2)</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\cos x)</td>
<td>(\sqrt{3}/2)</td>
<td>1/2</td>
<td>0</td>
<td>(-1)</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Periodicity: \(\sin(x + 2\pi) = \sin(x)\), \(\cos(x + 2\pi) = \cos(x)\), \(\tan(x + \pi) = \tan(x)\).

Identities: \(\sin^2 x + \cos^2 x = 1\), \(1 + \tan^2 x = \sec^2 x\), \(\sin(2x) = 2 \sin x \cos x\), \(\cos(2x) = \cos^2 x - \sin^2 x\).

Addition: \(\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y\) and \(\sin x \sin y = \cos(x) \cos y\).

Inverses: The range of \(\sin^{-1} x\), \(\tan^{-1} x\), and \(\csc^{-1} x\) is the subset of \([-\pi/2, \pi/2]\) avoiding values that correspond to \(x = \infty\). The range of the other inverse trig functions is a similar subset of \([0, \pi]\).

Exponentials and logarithms: \(a, b, t, u, y > 0\), \(r, v, w, x\) any real numbers: \(a^{v+w} = a^v a^w\), \(a^{vw} = (a^v)^w\), \(a^{-v} = 1/a^v\), \(a^0 = 1\), \((ab)^v = a^v b^v\), \(\log_a(t) = \ln(t)/\ln(a)\). \(e^x = y\) is equivalent to \(x = \ln y\), \(e^{\ln y} = y\), \(\ln(e^x) = x\). \(\ln(tu) = \ln(t) + \ln(u)\), \(\ln(u^v) = v \ln(u)\), \(\ln(1/u) = -\ln(u)\), \(\ln(1) = 0\), \(e \approx 2.718\).

Squeeze Theorem: If \(f(x) \leq g(x) \leq h(x)\) near \(x = a\) and \(\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L\), then \(\lim_{x \to a} g(x) = L\).

Intermediate Value Theorem: If \(f\) is continuous on \([a, b]\) and \(N\) is between \(f(a)\) and \(f(b)\), there is a number \(c\) in \([a, b]\) such that \(f(c) = N\). Corollary: If \(f\) changes sign from \(a\) to \(b\), then \(f(c) = 0\) with \(c\) between \(a\) and \(b\).

Definition of the Derivative: \(f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}\); \(f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}\).

\[
\begin{array}{c|c}
\text{Function} & \text{Derivative} \\
\hline
f(x) & f'(x) \\
\hline
c, \text{ const.} & 0 \\
x^r & rx^{r-1} \\
\ln x & \frac{1}{x} \\
e^x & e^x \\
\hline
\end{array}
\begin{array}{c|c}
\text{Function} & \text{Derivative} \\
\hline
f(x) & f'(x) \\
\hline
\sin x & \cos x \\
\cos x & -\sin x \\
\tan x & \sec^2 x \\
\sec x & \sec x \tan x \\
\cot x & -\csc^2 x \\
\csc x & -\csc x \cot x \\
\hline
\end{array}
\begin{array}{c|c}
\text{Function} & \text{Derivative} \\
\hline
f(x) & f'(x) \\
\hline
\sin^{-1}(x) & 1/\sqrt{1 - x^2} \\
\tan^{-1}(x) & 1/(x^2 + 1) \\
\sec^{-1}(x) & 1/(|x|/\sqrt{x^2 - 1}) \\
\cos^{-1}(x) & -1/\sqrt{1 - x^2} \\
\hline
\end{array}
\]

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Rules of Differentiation: $\frac{d}{dx}(cu) = c\frac{du}{dx}$, $c$ a const., or $(cf)’(x) = cf’(x)$. $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$, or $(f + g)’(x) = f’(x) + g’(x)$. **Product Rule:** $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$, or $(fg)’(x) = f(x)g’(x) + f’(x)g(x)$. **Quotient Rule:** $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u\frac{dv}{dx} - v\frac{du}{dx}}{v^2}$, or $(f/g)’(x) = (g(x)f’(x) - f(x)g’(x))/(g(x)^2)$.

Chain Rule: If $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{du}{dx} \frac{dy}{du}$, or $(f \circ g)’(x) = f’(g(x))g’(x)$. Replacing $x$ by $u$ and multiplying by $\frac{du}{dx}$, we can apply the Chain Rule to all boxed derivative formulas. Some examples are: $\frac{d}{dx}(u^n) = ru^{n-1} \frac{du}{dx}$, $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$, $\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}$, $\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$, $\frac{d}{dx}(\cos u) = -(\sin u) \frac{du}{dx}$, $\frac{d}{dx}(\tan u) = (\sec^2 u) \frac{du}{dx}$.

**Bodies in Free Fall:** If air resistance is neglected, then the height of a body in free fall near the surface of the earth is $s(t) = s_0 + v_0t - gt^2/2$, where $s_0$ is the position at time $t = 0$, $v_0$ is the velocity at time $t = 0$, and $g$ is the acceleration due to gravity with $g = 32\text{ft/s}^2$ or $g = 9.8\text{m/s}^2$.

**Linear or Tangent Line Approximation (or Linearization)** of $f(x)$ at $x = a$ is $L(x) = f(a) + f’(a)(x-a)$.

**Newton’s Method** to approximate a solution $r$ of $f(x) = 0$. Choose a point $x_0$ close to $r$. Calculate the terms $x_0, x_1, x_2, x_3, \ldots$ of the sequence defined recursively by $x_{n+1} = x_n - \frac{f(x_n)}{f’(x_n)}$.

**Rolle’s Theorem:** Suppose $f$ is a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval $(a, b)$. If $f(a) = f(b) = 0$, then $f’(c) = 0$ for some $c$ in $(a, b)$.

**Mean Value Theorem:** Suppose $f$ is a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval $(a, b)$. Then there is a point $c$ in $(a, b)$ such that $f(b) - f(a) = f’(c)(b-a)$.

**First Derivative Test:** Suppose that $f$ is a differentiable function and $f(c) = 0$. (a) If $f’$ changes sign from + to − at $x = c$, a local maximum occurs at $x = c$. (b) If $f’$ changes sign from − to + at $x = c$, a local minimum occurs. (c) If $f’$ does not change sign at $x = c$, neither a local maximum or minimum occurs at $x = c$.

**Second Derivative Test:** Suppose that $f$ is a twice differentiable function and $f’(c) = 0$. (a) If $f''(c) > 0$, a local minimum occurs at $x = c$. (b) If $f''(c) < 0$, a local maximum occurs. (c) If $f''(c) = 0$, the test fails.

**L'Hôpital’s Rule:** If $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\pm \infty}{\pm \infty}$, then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f’(x)}{g’(x)}$. (Here, $a$ may be a finite pt. or $\pm \infty$.)

**Integration or anti-differentiation:** $\int f(x) \, dx = F(x) + C$ means that $F’(x) = f(x)$. Formulas can be found by reversing the differentiation formulas: $\int x^r \, dx = x^{r+1}/(r+1) + C$, if $r \neq -1$ and $\int x^{-1} \, dx = \ln |x| + C$. 
