Formula Sheet for Math 151, Exam 1

Lines: If \((x_1, y_1), (x_2, y_2)\) lie on a line \(L\), the slope of \(L\) is \(m = \frac{y_2 - y_1}{x_2 - x_1}\) and the equation is \(y - y_1 = m(x - x_1)\).

Distance: \((x_1, y_1)\) to \((x_2, y_2)\): \(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\). Circle, center \((a, b)\), rad. \(r\): \((x - a)^2 + (y - b)^2 = r^2\).

In a right triangle: \(\sin \theta = \frac{opp}{hyp}\), \(\cos \theta = \frac{adj}{hyp}\), \(\tan \theta = \frac{opp}{adj}\), \(\sec \theta = \frac{hyp}{adj}\), \(\csc \theta = \frac{hyp}{opp}\), \(\cot \theta = \frac{adj}{opp}\).

Periodicity: \(\sin(x + 2\pi) = \sin(x)\), \(\cos(x + 2\pi) = \cos(x)\), \(\tan(x + \pi) = \tan(x)\).

Identities: \(\sin^2 x + \cos^2 x = 1\), \(1 + \tan^2 x = \sec^2 x\), \(2 \sin x \cos x = \sin(2x)\), \(\cos(2x) = \cos^2 x - \sin^2 x\).

Addition: \(\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y\), \(\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y\), \(\pi \approx 3.1416\).

Exponentials and logarithms: \(a, b, t, u, v, w, x\) any real numbers: \(a^u + w = a^v a^w\), \(a^{u+w} = (a^v)^w\), \(a^{-v} = 1/a^v\), \(a^0 = 1\), \((ab)^v = a^v b^v\), \(\log_a (x) = \log_b (x) / \log_b (a)\). \(e^x = y\) is equivalent to \(x = \ln y\), \(e^{\ln y} = y\), \(\ln(e^x) = x\), \(\ln(tu) = \ln(t) + \ln(u)\), \(\ln(u^v) = v \ln(u)\), \(\ln(1/u) = -\ln(u)\), \(\ln(1) = 0\), \(e \approx 2.718\).

Squeeze Theorem: If \(f(x) \leq g(x) \leq h(x)\) near \(x = a\) and \(\lim \_{x \to a} f(x) = \lim \_{x \to a} h(x) = L\), then \(\lim \_{x \to a} g(x) = L\).

Intermediate Value Theorem: If \(f\) is continuous on \([a, b]\) and \(N\) is any number between \(f(a)\) and \(f(b)\), there is a number \(c\) in \([a, b]\), such that \(f(c) = N\).

Corollary: If \(f\) changes sign from \(a\) to \(b\), then \(f(c) = 0\) with \(c\) between \(a\) and \(b\).

Definition of the Derivative: \(f'(x) = \lim \_{h \to 0} \frac{f(x + h) - f(x)}{h}\); \(f'(a) = \lim \_{x \to a} \frac{f(x) - f(a)}{x - a}\).

Rules of Differentiation: \(\frac{d}{dx}(cu) = c \frac{du}{dx}\), \(c\) a const., or \((cf)'(x) = cf'(x)\). \(\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}\), or \((f + g)'(x) = f'(x) + g'(x)\). Product Rule: \(\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}\), or \((fg)'(x) = f(x)g'(x) + f'(x)g(x)\).

Quotient Rule: \(\frac{d}{dx} \left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}\), or \((f/g)'(x) = \left(g(x)f'(x) - f(x)g'(x)\right)/(g(x)^2)\).

Chain Rule: If \(y = f(u)\) and \(u = g(x)\), then \(\frac{dy}{dx} = \frac{du}{dx} \frac{df}{du}\), or \((f \circ g)'(x) = f'(g(x))g'(x)\). Replacing \(x\) by \(u\) and multiplying by \(\frac{du}{dx}\), we can apply the Chain Rule to all boxed derivative formulas. Some examples are: \(\frac{d}{dx}(u^r) = ru^{r-1} \frac{du}{dx}\), \(\frac{d}{dx}(e^u) = e^u \frac{du}{dx}\), \(\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}\), \(\frac{d}{dx}(\sin u) = (\cos u) \frac{du}{dx}\), \(\frac{d}{dx}(\cos u) = -(\sin u) \frac{du}{dx}\), \(\frac{d}{dx}(\tan u) = (\sec^2 u) \frac{du}{dx}\).

Bodies in Free Fall: The distance above ground level of a body in free fall in the earth’s atmosphere is \(s(t) = s_0 + v_0 t - gt^2/2\), where \(s_0\) is the position at time \(t = 0\), \(v_0\) is the velocity at time \(t = 0\), and \(g\) is the acceleration due to gravity with \(g = 32\text{ft/s}^2\) or \(g = 9.8\text{m/s}^2\).