

Math 152, Spring 2008, Review Problems for Midterm 1

Your first exam is likely to have problems that do not resemble these review problems.

- (1) Calculate the work done against gravity when a column of height 12 feet and square base of side 2 feet is built out of a material with density 100 pounds per cubic foot.
- (2) Find the volume of a sphere of radius R in two different ways: (a) Using the Disk Method. (b) Using the Shells Method.
- (3) A triangular region \mathcal{R} is bounded by the lines $x = 2$, $y = 1$, $x + y = 2$. (a) Find the volume of the solid obtained by rotating \mathcal{R} about the x -axis. (b) Find the volume of the solid obtained by rotating \mathcal{R} about the y -axis.
- (4) Evaluate $\int \frac{3x^2 + 2x - 1}{(x - 1)(x^2 + 1)} dx$.
- (5) Evaluate $\int \frac{dx}{x\sqrt{36 - x^2}}$ and $\int \frac{dx}{(9 + x^2)^2}$.
- (6) Evaluate $\int_5^7 \frac{dx}{\sqrt{x - 5}}$ and $\int_2^\infty \frac{dx}{(2x - 1)(x - 1)}$.
- (7) Evaluate $\int (\ln x)^3 dx$ and $\int \sec^3 x dx$.
- (8) Evaluate $\int \tan^4 x \sec^4 x dx$ and $\int \tan^3 x \sec^3 x dx$.
- (9) Evaluate $\int \sec^{-1} x dx$.
- (10) Find N such that Simpson's Rule with N subintervals approximates $\int_2^{32} \ln x dx$ with Error ≤ 0.0001 .
- (11) Find the average of $\sin^4 x$ over the interval $[0, \pi/2]$.

If you evaluate an indefinite integral from this review sheet and find $\int f(x) dx = g(x) + C$, then you can check your answer $g(x) + C$ very easily using your calculator: Choose a random number x_0 and verify that $(g(x_0 + h) - g(x_0 - h))/(2h)$ approximates $f(x_0)$ for reasonably small values of h . This check uses the fact

$$\lim_{h \rightarrow 0} \frac{g(x_0 + h) - g(x_0 - h)}{2h} = g'(x_0).$$

If your answer $g(x) + C$ is correct, then $g'(x_0) = f(x_0)$ and

$$\lim_{h \rightarrow 0} \frac{g(x_0 + h) - g(x_0 - h)}{2h} = f(x_0).$$

You can also fix a small number h and compare the graphs of $(g(x + h) - g(x - h))/(2h)$ and $f(x)$. Of course, you cannot do these checks during an exam where calculators are not allowed.