Math 151, Fall 2010, Review Problems for Exam 1

Your first exam is likely to have problems that do not resemble these review problems.

(1) Describe the set \( S = \{ x \in \mathbb{R} : |2x - 4| > 2 \text{ and } |x - 3| \leq 1 \} \) in terms of intervals.

(2) Assume that \( f(x) \) is a function with domain \( \mathbb{R} \), and that \( f(x) \) is increasing on \([5, \infty)\). Explain why (a) and (b) must be true:
(a) If \( f(x) \) is an odd function then \( f(x) \) is increasing on \((-\infty, -5]\).
(b) If \( f(x) \) is an even function then \( f(x) \) is decreasing on \((-\infty, -5]\).

(3) Complete the square for \( 2x^2 - 8x - 10 \). Use your answer to find the minimum of \( 2x^2 - 8x - 10 \) and to solve \( 2x^2 - 8x - 10 = 0 \).

(4) Find functions \( f(x) \) and \( g(x) \) with domain \( \mathbb{R} \) such that \( f \circ g \neq g \circ f \).

(5) Find all solutions of \( 2 \sin^2 x = 1 + \cos(2x) \) in the interval \([0, 2\pi]\).

(6) Simplify \( \sec(\sin^{-1} x) \) and \( \cos(\tan^{-1} x) \).

(7) Solve \( \ln(x^2 + 7) - \ln(x^2 + 1) = 2 \ln 2 \).

(8) The position of a particle at time \( t \) (in seconds) is given by \( \frac{t}{1 + t^2} \) (in feet). Find the average velocity of the particle over the time interval \([1, 3]\).

(9) Find the exact values of the following limits. Do not use a calculator. Do not use L'Hôpital’s Rule, which appears much later in the textbook.

\[
\begin{align*}
\lim_{x \to 0} &\frac{x}{\sin(7x)} & \lim_{x \to 0} &\frac{\sin(5x)}{\sin(7x)} & \lim_{x \to 0} &\frac{x}{\tan x} & \lim_{x \to 0} &x \cos(x^{-3}) \\
\lim_{x \to 5^-} &\frac{x - 5}{|x - 5|} & \lim_{x \to 5^-} &\frac{x - 5}{|x - 5|} & \lim_{x \to 3^+} &\frac{x^2 - 20}{x^2 - 9} & \lim_{x \to 3^-} &\frac{x^2 - 20}{x^2 - 9} \\
\lim_{x \to 2} &\frac{x^2 + x - 6}{x^2 + 2x - 8} & \lim_{x \to 2} &\frac{x^3 - 2x^2 + x - 2}{x^3 - x^2 - x - 2} \\
\lim_{x \to 3} &\frac{4 - \sqrt{5x + 1}}{5 - \sqrt{8x + 1}} & \lim_{x \to 3} &\frac{4 - \sqrt{5x + 1}}{6 - 2x} & \lim_{x \to 0} &\frac{1 - \sec x}{x^2}
\end{align*}
\]

(10) Find constants \( a, b, c \) such that the function \( f(x) \), defined below, is continuous.

\[
f(x) = \begin{cases} 
ax^2 + b & \text{if } x \leq -1, \\
bx + c & \text{if } -1 < x < 1, \\
2c & \text{if } x = 1, \\
\frac{8}{1+x^2} & \text{if } 1 < x.
\end{cases}
\]

There are more problems on the next page.
(11) Explain why \( x = \cos x \) must have a solution.

(12) Use the \( \varepsilon, \delta \) definition of limit to prove \( \lim_{x \to 2} 3x + 4 = 10. \)

(13) Assume \( f(x) = x^{-2} \). Find \( f'(x) \) using the limit definition

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

of the derivative.

(14) A batter hits a pitched baseball. The height of the baseball is \(-16t^2 + 12t + 4\) feet at time \( t \) seconds after the bat meets the ball. An outfielder catches the ball when his glove is 6 feet above the ground. At what time did the fielder grab the baseball? What was the maximum height of the ball?

(15) A function \( f(x) \) is defined by

\[
f(x) = \begin{cases} 2x + 3 & \text{if } x < 1, \\ 3x + 2 & \text{if } x \geq 1. \end{cases}
\]

show that this function is continuous, but not differentiable.

(16) Find constants \( a, b \) such that the function \( f(x) \), defined below, is differentiable.

\[
f(x) = \begin{cases} 2x + 1 & \text{if } x < 2, \\ ax^2 + b & \text{if } x \geq 2. \end{cases}
\]

(17) Find the derivatives of the following functions of \( x \):

\[
(x^3 + x)^5 (1 + \cos x)^9 \quad \frac{\tan x}{1 + e^4x} \quad \sin \left(\sqrt{x^4 + x^2 + 3}\right) \quad \sec(e^x + \sqrt{x})
\]

(18) Find the second derivatives of the following functions of \( x \):

\[
(3 + x^{-3})^5 \quad \tan(7x) \quad \frac{1}{\sqrt{e^x + \cos x}} \quad e^{x^2 + 4x + 3}
\]

(19) Find the first, second, third and fourth derivatives of \( y = \cos(2x) \).

(20) Assume \( f(x) = e^{-x^2} \) and \( g(x) = \frac{1}{1 + x^2} \). Solve \( f''(x) = 0 \) and \( g''(x) = 0. \)